Math-M-Addicts Group A Entrance Exam 2016 – 2017

Student Name_____Grade in School_____

School Name_____Email_____

Instructions: A perfect solution to each problem below is worth 10 points. A score of 40 or more will guarantee admission to Group A; however, depending on the submitted tests, the cutoff for admission may be lower. That said, since we grade strictly, you should not limit yourself to 4 solutions, regardless of how clean you believe them to be; 4 solutions are often insufficient to obtain 40 points. Students must work on the problems on their own and not receive any help; students suspected of cheating will be disqualified. A reminder: this is a proof-based test for a proof-based class. Please write legibly and fully justify all your answers. Email your solutions to GroupASubmissions@mathmaddicts.com.

The submission deadline is Monday, September 12th, 2016.
Good luck!

Problem 1. Does equation $x^2 + y^2 + z^2 = 2xyz$ have any non-zero integer solutions?

Problem 2. Let x_1 and x_2 be the roots of quadratic polynomial $x^2 + px - \frac{1}{2p^2}$, where $p \neq 0$ is some real number. Find, with proof, the smallest possible value of $x_1^4 + x_2^4$ and determine for which p it is attained.

Problem 3. Suppose one can choose points R and T on sides BC and AD of a convex quadrilateral $ABCD$ so that $AR||CT$ and $DR||BT$. Prove $AB||CD$.

Problem 4. Numbers 1, 2, 4, 8, 16, 32, …, 512 are written on a whiteboard. One operation consists of taking some two numbers a and b, erasing them and replacing them by $\frac{ab}{a+b}$. Find, with proof, all possible values for the final number left after 9 of these operations.

Problem 5. Let real numbers $x_1, x_2, x_3, x_4, x_5, x_6$ satisfy $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 0$, and $x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2 = 6$. Prove $x_1x_2x_3x_4x_5x_6 \leq \frac{1}{2}$.

Problem 6. The kingdom of Mathlandia contains 2016 airports. Each airport is connected by a direct flight with at least 100 others. It's known that you can fly from each airport to any other, perhaps, with stopovers. Prove that, in fact, you can fly from any airport to any other making fewer than 65 connections.