Student Name: **School:** **Grade:**

Math-M-Addicts May 2018 Group I Entrance Exam

Instructions: Please write legibly and fully justify your answers. Points will be deducted for incomplete solutions. At the same time, it is OK to provide partial answers, as you may earn points for good ideas even if you do not have full solutions. Good luck!

Problem 1. The number 12 can be written as a sum of an integer and its smallest divisor greater than 1 in two different ways: 12 = 10+2 = 9+3. Find the smallest possible integer than can be represented as a sum of an integer and its smallest divisor greater than 1 in **FOUR** different ways.

Problem 2. Prove that for any integer n

$$1 * 2 * 3 + 2 * 3 * 4 + 3 * 4 * 5 + \ldots + n * (n+1) * (n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$

Problem 3. The fraction $\frac{3}{4}$ is written on the board. Every minute Fanny chooses two integers.

The first number, which is always between 80 and 100 (inclusive), is added to the numerator. The second number, which is between 100 and 120 (also inclusive), is added to the denominator. If at any point the numerator and denominator have a common factor, the fraction is reduced to its lowest terms. Can Fanny eventually get to the fraction $\frac{2}{3}$?

Problem 4. For which values of parameter r does equation

$$(r-3)x^2 - 2(r-2)x + r = 0$$

have 2 distinct real roots both greater than -1? Justify your answer.

Problem 5. Eleven white chairs are placed around a circular table. They are numbered 1 through 11 in increasing order. (For the avoidance of doubt, the chair order is 1, 2, ... ,10, 11 with 11 next to 1). In how many ways can Danielle paint some (or none) of the chairs red so that no three consecutive chairs are red?

Problem 6. Prove that the product of 100 consecutive positive integers can never be an exact 100th power of an integer.